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(article Melles-Milman, Lemma 4.5, p. 719-720, fascicule 4, 2006)

"Due to an error of the authors, a few lines were missing in the fourth paragraph of the original proof of Lemma IV.5 of our recently published paper, leaving a gap in the proof."

**Lemma 4.5.** — The same polynomials that generate $J_{(0,0)}$ also generate $J$ over a neighborhood of $(0,0)$ in $U \times \mathbb{C}^{n+1}$ and generate $I$ over a neighborhood of $\{0\} \times \mathbb{P}^n$ in $U \times \mathbb{P}^n$.

**Proof.** — Suppose that $J$ is generated in a neighborhood of $(0,0)$ by $F_1(x, y), ..., F_s(x, y)$, where $F_i(x, y)$ is a homogeneous polynomial of degree $d_i$ in $y$ with analytic coefficients in $x$. We will show that $I$ is generated on a neighborhood of $\{0\} \times \mathbb{P}^n$ in $U \times \mathbb{P}^n$ by the corresponding polynomials $F_i(x, \xi)$, where $[\xi] = [\xi_0 : ... : \xi_n]$ are homogeneous coordinates for $\mathbb{P}^n$. More precisely, we will show that $I$ is generated on a neighborhood of any point $q \in \{0\} \times \mathbb{P}^n$ by dehomogenizations of $F_1, ..., F_s$ near $q$.

Choose homogeneous coordinates $\xi$ on $\mathbb{P}^n$ such that $q = (0, [1 : 0 : ... : 0])$. Nonhomogeneous coordinates on the set $W = \{\xi_0 \neq 0\} \subset \mathbb{P}^n$ are $w_i = \frac{\xi_i}{\xi_0}$ for $1 \leq i \leq n$. We will check that $I$ is generated in a neighborhood of $q$ by the polynomials

$$\frac{F_i(x, \xi)}{\xi_0^{d_i}} = F_i\left( x, \frac{\xi}{\xi_0} \right) = F_i(x, 1, w_1, ..., w_n).$$

First we look at the maps $\sigma_1$ and $\sigma_2$ in local coordinates. We may use $(x, y_0, w)$ as local coordinates in $\sigma_2^{-1}(U \times W) \cong U \times \mathbb{C} \times W$. Local coordinates for $U \times \mathbb{C}^{n+1}$ are $(x, y_0, y_1, ..., y_n)$, where $y_i = y_0 w_i$ for $1 \leq i \leq n$. The maps $\sigma_1$ and $\sigma_2$ are given by

$$\sigma_1(x, y_0, w) = (x, y_0, y_0 w) \quad \text{and} \quad \sigma_2(x, y_0, w) = (x, w).$$

Suppose that $G$ is a holomorphic section of $I$ on a neighborhood of $q$ in $U \times \mathbb{P}^n$. Then $\sigma_2^* G$ is a holomorphic section of $\sigma_2^{-1} I$ in a neighborhood of $\sigma_2^{-1}(q) = \{(0, y_0, 0) : y_0 \in \mathbb{C}\}$. The homogeneous polynomials $F_1, ..., F_s$ that generate $J_{(0,0)}$ also generate $J = \sigma_1^* (\sigma_2^{-1} I)$ on a neighborhood of $(0,0) \in U \times \mathbb{C}^{n+1}$ (since $J$ is coherent, by the Direct Image Theorem), so their pullbacks $\sigma_1^* F_1, ..., \sigma_1^* F_s$ generate $\tilde{J} := \sigma_1^{-1} J$ on a neighborhood of $\sigma_1^{-1}(0,0) \in U \times \mathbb{C}^{n+1}$ and therefore generate $\tilde{I} := \sigma_2^{-1} I$ off
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\[ H := \sigma_1^{-1}(U \times \{0\}). \] Hence \( \text{Supp}(\tilde{I}/\tilde{J}) \subset H \) and therefore (by the complex analytic nullstellensatz) there exist an integer \( d > 0 \) and holomorphic functions \( A_1, \ldots, A_s \) on a neighborhood of the point \((x = 0, y_0 = 0, w = 0)\) in \( U \times \hat{\mathbb{C}}^{n+1} \) such that

\[ y_0^d \sigma_2^* G(x, y_0, w) = \sum_{i=1}^{s} A_i(x, y_0, w) \sigma_1^* F_i(x, y_0, w) \]

on that neighborhood. But \( \sigma_2^* G(x, y_0, w) = G(x, w) \) is independent of the value of \( y_0 \) and \( \sigma_1^* F_i(x, y_0, w) = F_i(x, y_0, y_0 w) = y_0^{d_i} F_i(x, 1, w) \) since \( F_i \) is homogeneous of degree \( d_i \) in \( y \). Therefore, by comparing terms in \( y_0^d \) of both sides of the equation above, it follows that there are holomorphic functions \( a_1, \ldots, a_s \) (on a neighborhood of \((x = 0, y_0 = 0, w = 0)\)) depending only on \( x \) and \( w \) such that

\[ G(x, w) = \sum_{i=1}^{s} a_i(x, w) F_i(x, 1, w). \]

Since the functions \( F_i(x, 1, w) \) are the local dehomogenizations of the homogeneous polynomials \( F(x, \xi) \), we are done. \( \square \)