ANNALES
DE LA FACULTÉ
DES SCIENCES
DE
TOULOUSE
Mathématiques

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KAWA Lecture Notes

Tome XXVII, n° 2 (2018), p. i-iv.

<http://afst.cedram.org/item?id=AFST_2018_6_27_2_r1_0>

Preface

This volume gathers lecture notes of some of the mini-courses delivered during the fifth, sixth and seventh editions of KAWA (Komplex Analysis Winterschool/Workshop and Applications).

In the last years, the research in Complex Analysis has moved from the classical questions of this subject to interact with other mathematical topics. This volume reflects this evolution, presenting lectures in complex dynamical systems, hyperbolic geometry, and interactions between geometric analysis, algebraic geometry and random point processes.

KAWA consists of a Winterschool and a Workshop with the aim of ensuring both a transmission of knowledge acquired over the last decades in Complex Analysis and training of young and senior researchers to these areas of interactions. The first four editions of KAWA were organized in: Toulouse&Albi (2010), Marseille at CIRM (2011), Barcelona (2012), Toulouse&Albi (2013), and lecture notes of some of the mini-courses delivered during those editions have been published in 2013 in the special volume of the *Annales de la faculté des sciences de Toulouse Sér. 6, Tome 22 no. 4*.

The fifth edition of KAWA took place in Marseille at CIRM (2014) jointly with NORDAN, the sixth KAWA was held in Pisa at the Centro De Giorgi (2015), and the seventh edition of KAWA was organized in Toulouse&Albi (2016), as part of the program *Thematic Semester Complex Geometry and Beyond*[(1)] . This volume consists of sets of lecture notes of those courses given at these last three editions of KAWA which were not already the object of a published article or survey.

The volume opens with the notes written by Valentino Tosatti, for the course *The Kähler–Ricci flow*, given during KAWA6 in Pisa in March 2015. The Ricci flow, first introduced by Hamilton in the eighties, is an evolution equation evolving a Riemannian metric by its Ricci curvature. The Ricci flow on a compact Kähler manifold X, starting at a Kähler metric ω_0, preserves the Kähler condition. The corresponding theory of the Kähler–Ricci flow has been intensively studied recently, the key feature being that the behavior of

[(1)] This thematic program took place within the activities of the LabEx CIMI (Centre International de Mathématiques et d'Informatique de Toulouse), see `http://www.cimi.univ-toulouse.fr/complex-geometry-and-beyond/`. More detailed information about the editions of KAWA can be found at `http://www.ub.edu/kawa7/`

the flow deeply reflects the geometric structure of the manifold X. When X is projective and the class $[\omega_0]$ is rational, the existence and long time behavior of the flow is intimately related to the Minimal Model Program in Algebraic Geometry.

In these lecture notes the author explains this conjectural picture, and proves several results which go somehow towards achieving this picture.

The second contribution is the paper written by Tom Carroll, Jordi Marzo, Xavier Massaneda and Joaquim Ortega-Cerdà as a reference for the course *Fekete points, an overview* given by Joaquim Ortega-Cerdà during KAWA6 in Pisa in March 2015. The paper deals with a particular kind of random point process on complex projective manifolds known as β-ensembles. This is a random point process on a polarized compact complex manifolds (X, L) distributed according to the β power of a determinant of sections of the positive line bundle L. The authors give a precise estimate of the rate of convergence of the empirical measure associated to a β-ensemble to its limiting measure.

A particular case is the spherical ensemble of generalized random eigenvalues of pairs of matrices with independent identically distributed Gaussian entries. An interesting situation occurs when $\beta \to \infty$. In this case the probability charges the maxima of the determinant function. Such a set of points is known as a Fekete sequence of points. The distribution of these sequences has been intensively studied in the past, in particular in a cornerstone work of Berman, Boucksom and Witt-Nyström establishing the equidistribution of Fekete points with respect to the equilibrium measure.

The third contribution consists in the lecture notes written by Laura DeMarco for the course *Dynamical moduli spaces and elliptic curves* given during KAWA6 in Pisa in March 2015. The reader will find in these notes, a presentation of a connection between the complex dynamics of a family of rational functions $f_t \colon \mathbb{P}^1 \to \mathbb{P}^1$, parameterized by t in a Riemann surface X, and the arithmetic dynamics of f_t on rational points $\mathbb{P}^1(k)$ where $k = \mathbb{C}(X)$ or $\overline{\mathbb{Q}}(X)$. Laura DeMarco explains an explicit relation between stability and canonical height, giving a proof containing part of the Mordell–Weil theorem for elliptic curves over function fields. Several questions and conjectures about these families are presented guided by the principle of "unlikely intersections" from arithmetic geometry.

The fourth contribution are the lecture notes written by Erwan Rousseau, for his course *Complex Hyperbolic Geometry*, given during KAWA5-Nordan at CIRM in Marseille in March 2014, concerning the study of hyperbolicity properties of complex algebraic varieties. These notes nicely survey some of the most important approaches to this active research area, presenting

the most recent results on the subject due to Brotbek, Brotbek–Darondeau, Deng and Xie on the Kobayashi and Debarre conjectures.

The author proves McQuillan's degeneracy theorem for smooth foliations. He then introduces jet spaces and shows that the Green-Griffiths locus can be equal to the whole space. The survey ends with a section devoted to the author's results on the Kobayashi hyperbolicity, modulo the boundary, of quotients of bounded symmetric domains.

The volume closes with the lecture notes written by Xavier Buff for the course on *Wandering domains for polynomials*, given during KAWA7 in Toulouse in March 2016. Given a polynomial $f\colon \mathbb{C} \to \mathbb{C}$, its filled-in Julia set K_f is the set of points with bounded orbit under iteration of f and in 1985 Sullivan proved the so-called No Wandering Domain Theorem, stating that every connected component of the interior of K_f is eventually periodic. In these notes, Xavier Buff presents a proof due to Adam Epstein based on a density theorem due to Bers for quadratic differentials. Then, following an approach based on parabolic implosion, and initially suggested by Misha Lyubich and Han Peters, he explains the key steps leading to the first example showing that Sullivan's result does not hold for polynomials $F\colon \mathbb{C}^2 \to \mathbb{C}^2$.

Acknowledgements

First of all, the continued existence of the KAWA series wouldn't have been possible without the participation of dozens of people from all around the world (from 60 to 100 for each edition), and the support of the advisors at various universities who encouraged their students to go and found them funding: their enthusiasm and trust deserves our first thanks.

We are also thankful to all the speakers during the winterschools and the workshops, and to all the others who offered to give a talk, but couldn't be included in our programme, severely constrained by time. A special thank goes to the the scientific committees, who helped and guided us through the difficult process of selecting which talks would be given, and to the local organizers for their priceless work.

We are grateful for the financial support from various sources which enabled us to subsidize, in particular, part of the expenses of most of the younger participants in the various editions of KAWA: our institutions, the Institut de Mathématiques de Toulouse, the Université Paul Sabatier, the Universitat de Barcelona; the Institut de Matemàtica de la Universitat de Barcelona (IMUB), the Universitá di Pisa, the LabEx CIMI, the Agence

Nationale de la Recherche (through its MACK and LAMDA projects), the Institut Universitaire de France, the CNRS, the CIRM, the European Mathematical Society.

Last but not least, we wish to thank the Annales de la Faculté des Sciences de Toulouse for offering us the opportunity to have this second special issue devoted to KAWA, the anonymous referees, and the authors of this volume who, in addition to preparing the courses they gave, put in the extra work to write up the notes presented here.

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